A Journey to Europa 🚀

IJSO Theory Mock Test

Solutions

Problem 1—Gaseous Planets (5.25 points)

Part A: Diamond Formation

A1. Find the enthalpy change of reaction.

(0.25 points)

Calculation:

$$\Delta_r H = \Delta_f H(\text{diamond}) - \Delta_f H(\text{graphite}) = 1.9\,\frac{\text{kJ}}{\text{mol}}$$

$$\Delta H = \mathbf{1.9\,\frac{kJ}{mol}}$$

A2. Find the activation energy E_{a_1} of reaction (1)

(1.00 points)

Calculation:

$$k_{1,2} = Ae^{-\frac{E_{a1}}{RT_{1,2}}}$$

(0.25 points)

Dividing the two equations, $r = e^{\frac{E_{a1}}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$

(0.50 points)

Finally, we get $E_{a1} = \frac{R\ln r}{\frac{1}{T_2}-\frac{1}{T_1}} = 538.4\text{kJ}$ for the forward reaction

(0.25 points)

E_{a_1} = $\mathbf{538.4kJ}$

A3. Find the activation energy E_{a_2} of the inverse reaction.

(0.25 points)

Calculation:

Since the energy level of the products are higher than the energy levels of the reactants by H, the energy required for the inverse reaction to occur must be given by:

$$\Delta H + E_{a2} = E_{a1} \Leftrightarrow E_{a2} = E_{a1} - \Delta H = 536.5\,\frac{kJ}{mol}$$

E_{a_2} = $\mathbf{536.5\,\frac{kJ}{mol}}$

Part B: Decomposition of methane

B1. Calculate the enthalpy change of reaction (2)

(0.25 points)

Calculation:

$$\Delta_f H(\text{graphite}) = \Delta_f H(H_2) = 0$$

$$\Delta H = \Delta_f H(CH_4) = -74.2\,\frac{kJ}{mol}$$

Alternatively, the enthalpy change of reaction must be, by definition, the enthalpy of formation of CH_4, since reaction (2) is the reaction of CH_4 formation from the most stable form of its chemical elements.

$\Delta H = \mathbf{-74.2\,\frac{kJ}{mol}}$

B2. Which of the following will increase the rate of the forward reaction? *Tick the appropriate boxes.*

(0.50 points)

- ☐ Increase the temperature.
- ✓ Decrease the temperature.
- ☐ Increasing the pressure.
- ✓ Decreasing the pressure.
- ☐ Adding more methane.
- ✓ Adding a catalyst.

B3. Find the equilibrium concentration of hydrogen gas $[H_2]$ in mol/L.

(0.75 points)

Calculation:

$$\mathrm{K} = \frac{[\mathrm{CH_4}]}{[\mathrm{H_2}]^2[\mathrm{C}]}$$

(0.25 points)

Graphite is solid, so its concentration in equilibrium calculations can be considered equal to 1.

(0.25 points)

$$[\mathrm{H_2}] = \sqrt{\frac{[\mathrm{CH_4}]}{\mathrm{K}}} = 0.63\,\frac{\mathrm{mol}}{\mathrm{L}}$$

(0.25 points)

$$[\mathrm{H_2}] = 0.63\,\frac{\mathrm{mol}}{\mathrm{L}}$$

Part C: Reaction in the Planets

Calculate the enthalpy change of reaction (3)

(0.25 points)

Calculation:

$$\Delta H = \Delta_f H\ (\text{diamond}) - \Delta_f H\ (CH_4) = 76.1\,\frac{kJ}{mol}$$

$$\Delta H\ = 76.1\,\frac{kJ}{mol}$$

Part D: Methane combustion

D1. Write the balanced equation for this reaction.

(0.30 points)

$$\mathbf{CH_4 + O_2 \longrightarrow CH_2O + H_2O}$$

D2. Find the values of constants a and b.

(0.30 points)

Calculation:

$$K = \frac{[CH_2O][H_2O]}{[CH_4][O_2]}$$

(0.10 points)

All units cancel out, so K is adimensional. That means $a = b = 0$

(0.20 points)

a = **0**

b = **0**

D3. What is the total mass of the gases inside the reactor?

(0.50 points)

Calculation:

The total amount of gases is $n = \frac{PV}{RT} = 0.012\text{mol}$. But the mixture is equimolar, so we have 0.006mol of each gas.

(0.30 points)

The total mass is $m = 0.006 \cdot 32 + 0.006 \cdot 16 = 0.288\text{g}$

(0.20 points)

Mass = 0.288g

D4. What is the value K_0 of the equilibrium constant?

(0.60 points)

Calculation:

The number of moles of methane in the equilibrium mixture is given by the ideal gas law, $n = \frac{pV}{RT} = 1.2 \cdot 10^{-16}\ mol$. Because the initial mixture was stoichiometric, the amount of oxygen at equilibrium is also $1.2 \cdot 10^{-16}\ mol$.

(0.20 points)

We can consider that almost all the methane (and oxygen) were converted into formaldehyde and water, so $n_{H_2O} = n_{CH_2O} \cong 0.006mol$

(0.10 points)

$$K_0 = \frac{n_{H_2O}/V \cdot n_{CH_2O}/V}{n_{CH_4}/V \cdot n_{O_2}/V} = \frac{n_{H_2O} \cdot n_{CH_2O}}{n_{CH_4} \cdot n_{O_2}} = 2.5 \cdot 10^{27}$$

(0.30 points)

K_0 = $\mathbf{2.5 \cdot 10^{27}}$

D5. Compare each of the values of constants K_A, K_B and K_C with the initial K_0 value. Put > or = or <

(0.30 points)

K_A	=	K_0
K_B	<	K_0
K_C	=	K_0

Neither the inverse, nor the forward reaction change the number of moles of gas. So, a change in pressure won't modify the equilibrium position

If the reaction is exothermic, a rise in temperature will move the equilibrium to the left. This will result in a decrease of the equilibrium constant.

Catalysts only affect reaction rate, not equilibrium.

Problem 2—Before the Journey in Europa (3.85 points)

Part A: Preparing the Fuel Supply

A1. How many grams of hydrogen gas do we need?

(0.60 points)

Calculation:

Number of moles $n = \frac{10^6 \text{ kJ}}{286 \text{ kJ/mol}} \approx 3496.5 \text{ mol}$

(0.30 points)

The mass is $3496.5 \text{ mol} \cdot 2 \frac{\text{g}}{\text{mol}} = 6993\text{g}$

(0.30 points)

Mass = **6993g**

A2. What is the minimum tank volume (in liters) needed to store all the hydrogen?

(0.65 points)

Calculation:

$V = \frac{nRT}{p} = 431.7\text{L}$

V_{min} = **431.7L**

A3. Calculate the extra tank volume (in liters) needed to store the oxygen needed for the entire trip if the trip lasts for 2 weeks.

(0.60 points)

Calculation:

Total energy needed = $12000 \times 14 \times 10 = 1.68 \times 10^6\ J$

No of oxygen moles = $\frac{1.68\times10^6\ \text{J}}{286\times1000\times2} = 2.937$ moles

V = $\frac{nRT}{P} = \frac{2.937\times0.08206\times300}{200} = 0.362\text{L}$

$V_{\text{extra}} = 0.362$ L

Part B: Thermal Shielding & Material Selection

B1. What is the heat flux (W/m²) through the aluminum if one side is at 25°C and the other at –160°C?

(0.50 points)

Calculation:

$\Phi = \text{KA}\frac{\Delta\text{T}}{\Delta\text{x}}$

(0.20 points)

The heat flux is $\frac{\Phi}{\text{A}} = \text{K}\frac{\Delta\text{T}}{\Delta\text{x}} = 3.79 \cdot 10^6\,\frac{\text{W}}{\text{m}^2}$

(0.30 points)

Heat Flux = $\mathbf{3.79 \cdot 10^6\,\frac{W}{m^2}}$

B2. Calculate the new total heat flux through the combined system (aluminum + foam), treating the layers as resistors in series.

(1.00 points)

Calculation:

Assuming the temperature at the aluminum - polyurethane foam is T, the thermal conductivity would be

$$\frac{\Phi}{\mathrm{A}} = \mathrm{K}\frac{\Delta \mathrm{T}}{\Delta \mathrm{x}} = 205\frac{T+160}{0.01} = 0.03\frac{25-T}{0.02}$$

Solving, we get T = -159.986

Flux = $277.48\,\frac{W}{m^2}$

Heat Flux = $277.48\,\frac{W}{m^2}$

B3. Calculate the total energy absorbed (in joules) by the solar panel due to sunlight throughout the whole trip.

(0.50 points)

Calculation:

Power per unit area in Europa = $\frac{3.8\cdot 10^{26}}{4\times \pi \times (7.80\cdot 10^{8})^2} = 4.97\cdot 10^{7}\ W/m^2$

Power in solar cell = $4.97\cdot 10^{7} W/m^2 \times 0.5\ m^2 \times 0.85 = 2.11\times 10^{7} W$

Total energy = $2.11\times 10^{7}\ W \times 5\times 24\times 3600 = 9.12\times 10^{12}\ J$

Energy = $9.12\times 10^{12}\ J$

Extra space for problem 2:

Problem 3—Discovery during the Journey (4.25 points)

Part A: Chemical Analysis of the Reddish Sample

A1. If 1.80 g of $Fe(OH)_2$ precipitate is formed, calculate the mass of $FeSO_4$ originally present.

(0.50 points)

Calculation:

$FeSO_4$ + 2NaOH → $Fe(OH)_2$↓ + Na_2SO_4

Moles of $Fe(OH)_2$ = $\frac{1.80\ g}{89.85\ g/mol} = 0.0200\ mol$

$FeSO_4$ moles = $Fe(OH)_2$ moles = 0.0200 mol

$FeSO_4$ mass = 0.0200 mol × 151.91 g/mol = 3.038 g

Mass = 3.038 g

A2. If 0.875 g of $Mg(OH)_2$ is obtained, calculate the mass of $MgCl_2$ in the original sample.

(0.50 points)

Calculation:

$MgCl_2$ + 2NaOH → $Mg(OH)_2$↓ + NaCl

Moles of $Mg(OH)_2$ = $\frac{0.875\ g}{58.32\ g/mol} = 0.0150\ mol$

Moles of $MgCl_2$ = $0.0150\ mol \times 95.21\ g/mol = 1.428\ g$

Mass = $1.428\ g$

A3. Based on your answers to A1 and A2, calculate the mass percent composition of $FeSO_4$ and $MgCl_2$ in the original 10.0 g sample.

(0.25 points)

Calculation:

Mass percent of $FeSO_4 = \frac{3.038\ g}{10.0\ g} = 30.38\ \%$

Mass percent of $MgCl_2 = \frac{1.428\ g}{10.0\ g} = 14.28\ \%$

Mass percent of $FeSO_4$ = 30.38 %

Mass percent of $MgCl_2$ = 14.28 %

Part B: The depth of the crater

B1. What is the gravitational acceleration on Europa?

(0.50 points)

Calculation:

$$h = \frac{1}{2}gt^2 => g = \frac{2h}{t^2} = 1.32 m/s^2$$

g = $1.32 m/s^2$

B2. What is the speed of sound?

(0.75 points)

M = 32g/mol =0.032 kg/mol, (molar mass of O2), plugging in the values of the temperature, molar mass, the gas constant R and γ in correct units it is possible to find v = 209m/s

v = 209m/s

B3. What is the depth of the crater?

(1.50 points)

Calculation:

$$t = \sqrt{\frac{2h}{g}} + \frac{h}{v}$$

$$(t - \frac{h}{v})^2 = \frac{2h}{g}$$

$$\frac{h^2}{v^2} - 2h(\frac{1}{g} + \frac{t}{v}) + t^2 = 0$$

Solving the quadratic equation for h and rounding it to 3 significant figures, the right solution is $h = 140m$, plugging in the other solution (somewhere around 72 km) into the initial equation will not yield a time of 15.21 seconds, it is also too large, so it does not make sense.

Depth = $140m$

B4. What is the wavelength of that sound wave?

(0.25 points)

Calculation:

$$v = \lambda f \;\; => \;\; \lambda = \frac{v}{f} = 35.8\, mm$$

Wavelength = $35.8\, mm$

Extra space for problem 3:

Problem 4—Life in Europa (9.40 points)

Part A: Interpreting Signs of Life

A1. Identify two of these gases. Put a cross in the appropriate boxes.

(0.50 points)

Oxygen	X
Water Vapor	
Methane	X
Carbon Dioxide	

A2. *Tick the appropriate box.*

- ☐ Proteins with specific chirality (e.g., L-amino acids)
- ☐ Nucleic acids (like DNA or RNA)
- ☐ Polysaccharides (e.g., storage or structural polymers)
- ✓ Hydrocarbons or amino acids.

(0.30 points)

A3. *Tick the appropriate box.*

- ☐ Greater than
- ✓ Equal to
- ☐ Less than
- ☐ Has no effect

(0.30 points)

Part B: Life in the European Ocean Depths

B1. a) Calculate the concentration of these structures in terms of structures per mL.

(0.40 points)

Calculation:

250 rods / 0.010 ml = 25 000 rods/ml

Structures per milliliter = 25 000

B1. b) Calculate the approximate volume of a single average rod-shaped structure in cubic micrometers (μm^3).

(0.50 points)

Calculation:

$2.0\ \mu m \times (0.5\ \mu m)^2 \times \pi = 1.6\ \mu m^3$

Volume = $1.6\ \mu m^3$

B1. c) What is the volume of a single structure in liters?

(0.30 points)

$1.6\ \mu m^3 / 10^{15} = 1.6 \times 10^{-15}\ L$

Volume = $1.6 \times 10^{-15}\ L$

B2. *Tick the appropriate boxes (continuation on next page):*

- ✓ Cell membranes with specialized lipids to maintain fluidity under high pressure and low ambient temperatures (away from direct vent heat).
- ✓ Enzymes that function optimally at near-freezing temperatures (psychrophilic) for life further from vent openings.
- ✓ Enzymes that function optimally at near-freezing temperatures (psychrophilic) for life further from vent openings.

- ✓ Efficient mechanisms to repair DNA damage, considering Jupiter's radiation environment and potential residual radiation.
- ☐ Strict dependence on sunlight for energy production.
- ✓ Ability to utilize inorganic chemical compounds as an energy source.
- ☐ Thick cell walls primarily composed of cellulose for structural support.

(0.60 points)

B3. (Fill-in-the-blank)

To maintain cell membrane fluidity in the generally cold deep ocean of Europa (away from the immediate vent plume), the fatty acid chains in their phospholipid bilayer membrane would likely need a **higher** (higher / lower) proportion of chemical bonds that introduce kinks into the fatty acid chains, such as **double** (double / triple / single) bonds.

(0.40 points)

Part C: Metabolism and the Chemistry of Alien Life

C1. a) What is the general metabolic term for organisms that obtain energy by oxidizing inorganic substances, like the Europan microbes utilizing H_2S?

(0.30 points)

Term = chemo (auto) (litho) trophs

C1. b) In the given reaction, is H_2S acting as an oxidizing agent or a reducing agent? Put a cross in the appropriate box.

(0.20 points)

Oxidizing Agent	
Reducing Agent	X

C2. How much energy (in kJ) does this colony generate per hour from this reaction? (0.40 points)

Calculation:

$$\frac{0.64\ \text{g}}{32\ \text{g/mol}} = 0.02\ \text{mol}$$

$$0.02\ mol\ \times\ 210\ kJ/mol\ =\ 4.2\ kJ$$

4.2 kJ / 12 h = 0.35 kJ / h

Energy = 0.35 kJ

C3. a)

- ✓ **(Tick):** If the statement is scientifically correct AND directly relevant to the context.
- **0 (Zero):** If the statement is scientifically correct BUT its direct relevance to the context is minor or indirect, or it pertains to a different aspect not central to the discussion.
- ✗ **(Cross):** If the statement is scientifically incorrect.

Statements	Evaluation
A. The tendency of Group 14 elements to form extended chains by bonding with themselves (catenation) is a key factor in their ability to create the framework of macromolecules.	✓
B. Due to possessing four valence electrons, elements in Group 14 typically engage in forming up to four covalent bonds with other atoms.	✓

C. Certain microorganisms, like diatoms, incorporate silicon into their cell walls in the form of silica, creating intricate protective structures.	0
D. When forming the structural basis of large molecules, Group 14 elements achieve stability primarily through the formation of ionic bonds.	✗
E. The energy and stability of the bonds formed between identical atoms of a Group 14 element (e.g., C-C vs. Si-Si) are virtually the same, making them equally suitable for chain formation under all conditions.	✗

(0.50 points)

C3. b) *Tick the appropriate box.*

- ☐ Liquid methane (a non-polar solvent abundant in the outer solar system)
- ☐ Highly purified water (as it is a universal solvent)
- ✓ Molten sulfur (found in volcanic regions) could dissolve some silicon compounds
- ☐ Gaseous hydrogen (as a lightweight atmospheric component)

(0.40 points)

c) Fill in the blanks. Biological catalysts and many cellular structures on Earth are primarily made of macromolecules called proteins. These are polymers of **amino acids.** For any molecule to effectively serve as the primary hereditary material (e.g., DNA on Earth), it must primarily be capable of accurate self-**replication** and the stable **storage** of genetic information. A major challenge for hypothetical silicon-based life (compared to carbon-based life that produces gaseous CO_2 as a waste product) is that the primary oxide of silicon (silicon dioxide, SiO_2) is typically a **solid** (physical state) at common planetary surface temperatures, making its metabolic cycling and disposal difficult.

(0.60 points)

Part D: Dynamics of Europan Microbial Life

D1. a) Assuming continuous exponential growth, calculate the number of microbial cells expected after 40 Earth hours.

(0.70 points)

Calculation:

40 h / 10 h = 4

$2.0 \times 10^4 \times 2^4 = 3.2 \times 10^5$

Number of Microbial Cells = 3.2×10^5

D1. b) How many generations would have occurred during these 40 Earth hours?

(0.30 points)

Calculation:

40 h / 10 h = 4

No. of Generations = 4

D2. a) Calculate the frequency of the T_S allele in this sampled population.

(0.60 points)

Calculation:

$$\frac{60 \times 2 + 80}{200 \times 2} = 0.5$$

Frequency of the T_S allele = 0.5

D2. b) Calculate the frequency of the T_F allele in this sampled population.

(0.60 points)

Calculation:

$$\frac{60 \times 2 + 80}{200 \times 2} = 0.5$$

Frequency of the T_F allele = 0.5

D2. c) what would be the expected frequency of the $T_F T_S$ (heterozygous) genotype? Show your calculation.

(0.60 points)

$$2 \times 0.5 \times 0.5 = 1 \times 0.5 = 0.5$$

$T_F T_S$ frequency = 0.5

D2. d) *Tick the appropriate boxes.*

- ☐ Heterozygous genotypes are more suited since they have both characters.
- ✓ Lacking distinct specializations reduces an organism's fitness in environments that specifically favor any one of those specializations.
- ☐ The T_S and T_F alleles are codominant, and codominance inherently causes a disruption in the expected Hardy-Weinberg equilibrium for heterozygotes.
- ☐ The population at Vent Prime is small and relatively isolated, leading to a higher incidence of inbreeding.
- ✓ Vent Prime may consist of distinct micro-habitats with differing selective pressures.

(0.60 points)

D3. e) what term describes the status of this allele in the population?
(0.30 points)

Term = **fixated**

Extra space for problem 4:

Problem 5—Life-Sustaining Chemicals (2.25 points)

A. If in the sample carbon is only found as one isotope and oxygen as two isotopes, find the mass numbers of the isotopes.

(0.50 points)

Calculation:

Let $A_{1,2}$ be the mass number of the oxygen isotopes and A_3 the mass number of the carbon isotope.

We have $2A_1 + A_3 = 44$, $A_1 + A_2 + A_3 = 46$, $2A_2 + A_3 = 48$

(0.15 points)

Solving the system gives $A_1 = 16$, $A_2 = 18$, $A_3 = 12$

(0.35 points)

(0.40 points in total if no proof is given, just the values)

Mass number of the 1st oxygen isotope: **16**
Mass number of the 2nd oxygen isotope: **18**
Mass number of the carbon isotope: **12**
B. Find the number of neutrons in each isotope.

(0.25 points)

Calculation:

Mass number = no. of protons + no. of neutrons

Atomic number = no. of protons

no. of neutrons = mass number – atomic number

(0.15 points for formula)

(0.10 points for values)

Number of neutrons in the 1st oxygen isotope: **8**
Number of neutrons in the 2nd oxygen isotope: **10**
Number of neutrons in the carbon isotope: **6**

C. Find the relative abundances of the three isotopologues.

(0.25 points)

Calculation:

The relative abundance of each isotopologue is proportional to the peak intensity.

Because the sum of all abundances is 1, the abundance of an isotopologue is given by $\alpha_i = \frac{I_i}{I_{44}+I_{46}+I_{48}}$

(0.15 points)

Substituting values, $\alpha_{44} = 0.81$, $\alpha_{46} = 0.18$, $\alpha_{48} = 0.01$ (percentages are also acceptable)

(0.10 points)

Relative abundance of 44u isotopologue = **0.81**

Relative abundance of 46u isotopologue = **0.18**

Relative abundance of 48u isotopologue = **0.01**

D1. Attribute each abundance to its isotopologue.

(0.50 points)

Isotopologue	Abundance
44u isotopologue	$\mathbf{a_1^2}$
46u isotopologue	$\mathbf{2 \cdot a_1 \cdot a_2}$
48u isotopologue	$\mathbf{a_2^2}$

The abundance of the isotopologue containing only the ^{16}O isotope will only depend on its abundance (a_1). Similarly, the abundance of the isotopologue containing only ^{18}O will only depend on a_2, while the abundance of the isotoplogue containing both isotopes will depend on both a_1 and a_2.

D2. Calculate the abundances of the two isotopes in the sample from Europa.

(0.75 points)

Calculation:

$\alpha_{44} = a_1^2 \Leftrightarrow a_1 = \sqrt{\alpha_{44}} = 0.9$

(0.50 points)

$a_2 = 1 - a_1 = 0.1$

(0.25 points)

Any other abundance/pair of abundances can be used to get to this result.

Isotope	Abundance
1st isotope	**90%**
2nd isotope	**10%**

Extra space for problem 5:

Problem 6—Jicu and His Rover (5.00 points)

Part A: Uncontrolled Descent

A1. Assume that Jicu weighs 70 kg and that the rover weighs 500 kg. Find how much force is pulling the rover downhill with Jicu onboard.

(0.25 points)

Calculation:

The value for the gravitational acceleration obtained on part B1 of problem 3 is $g = 1.32\ m/s^2$. The component of the gravitational force parallel to an inclined plane with slope θ is $F_{//} = mgsin\theta$ where m is the total mass, which is the sum of Jiu's mass with the rover's mass (500kg + 70kg = 570kg)

Plugging in the values on the expression above gives:

$F_{//} = (70kg + 500Kg) \times 1.32m/s^2 \times sin(15°) = 195\ N$

Force = 195 N

A2. Assuming no brakes and no slipping, calculate the acceleration of the rover.

(0.25 points)

Calculation:

$a = F_{//}/m = 195N/570kg = 0.342\ m/s^2$.

Alternatively, you could also multiply the gravity g by the slope

$a = gsin\theta = 1.32m/s^2 \times sin(15°) = 0.342\ m/s^2$

Acceleration = 0.342 m/s^2

A3. How much braking torque per wheel is needed to maintain constant speed?

(0.50 points)

Calculation:

To maintain constant speed, there must be no acceleration. So, the force due to the breaking torques must have the same magnitude as the force pulling the rover downhill, since there are 4 wheels

$4F = F_{//} = 195N$ => $F = 48.7\ N$

The breaking torque on a single wheel with radius R, with a breaking force F is

$\tau = F \times R = 48.7\ N \times 0.30\ m = 14.6\ Nm$

Torque = 14.6 Nm

A4. Calculate the thermal energy generated by friction over 4 m (along the slope) of uncontrolled slide in the rough patch.

(0.50 points)

Calculation:

$F_f = \mu_k\ mg\ cos\theta$

So, the work done by friction is:

$|W_f| = \mu_k \times mg\ cos\theta \times d = 0.15 \times 570kg \times 1.32m/s^2 \times cos15° \times 4m$

$|W_f| = 436\ J$

So, the thermal energy generated by friction is 436 J

Energy generated = 436 J

A5. Estimate the energy dissipated during the impact (in kJ).

(0.50 points)

Calculation:

The energy dissipated during the impact is the kinetic energy the rover had right before the collision

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 570kg \times (3.2m/s)^2 = 2918,4\,J$$

$$E = 2.92\ kJ$$

Energy = $2.92\ kJ$

A6. Calculate the rotational kinetic energy lost.

(0.50 points)

Calculation:

All the potential energy gets converted into rotational kinetic energy, so the total rotational kinetic energy lost is the potential energy it had at the moment of the flip.

$$E = mgh = 570\ kg \times 1.32\ m/s^2 \times 1.2m = 903\ J$$

Kinetic Energy Lost = $903\ J$

Part B: Aftermath of the Crash

B1. Calculate whether the frame would have deformed plastically or remained intact.

(1.00 points)

Calculation:

The total impulse exerted by the snow is the change momentum, therefore

$$F \times \Delta t = mv - 0$$

$$F = \frac{mv}{\Delta t} = \frac{570\,kg \times 3.2\,m/s}{0.8\,s} = 2280\,N$$

Dividing by force by the area gives

$$2280N/0.5m^2 = 4560\,N/m^2$$

Which is way smaller than the compressive yield force, so the frame will remain intact.

B2. Calculate the spring constant of the rover's suspension system based on the observed vibrations.

(0.50 points)

Calculation:

The oscillation frequency for a spring with mass m and spring constant k is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

So, the spring constant is

$$4\pi^2 f^2 m = 4\pi^2 \times (4Hz)^2 \times 200\ kg = 1.26 \times 10^5\ N/m$$

Spring constant = $1.26 \times 10^5\ N/m$

B3. How much force is required to overcome this static friction if Jicu attempts to move the rover uphill at a constant speed?

(0.75 points)

Calculation:

To move uphill at constant speed, the applied force F must balance both gravity and friction:

$$F = mg \sin\theta + \mu mg \cos\theta$$

$$F = 570\ kg \ \times\ 1.32\ \times\ (\sin 1\ 2^\circ + 0.15 \cos 1\ 2^\circ)$$

$$F\ =\ 266.83\ N$$

Force required = $266.83\ N$

B4. How much time (in minutes) will they have before the battery runs out?
(0.25 points)

Calculation:

Total power output = 80 W + 50 W = 130 W

Total energy available = 780,000 J

Total time = $\frac{780{,}000\ \text{J}}{130\ \text{W}} = 6000\ s = 100\ min$

Time = 100 min